Web Appendix to: "Social Network Formation and Labor Market Inequality"*

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1 Web Appendix

1.1 Lemma 1

Lemma 1. *The optimal number of links follows a power law distribution with pdf:*$g(t) = \frac{1}{\Delta c t^2}\frac{(1-\delta)}{\delta E[k]}E[1-(1-\delta)^k]wp(1-p)$, *where* $E[k] = \sqrt{\frac{(1-\delta)}{\Delta c\delta}E[1-(1-\delta)^k]wp(1-p)\ln(N)}$ *and* $E[1-(1-\delta)^k] = \frac{1}{1+\frac{p(1-p)(1-\delta)wA}{\Delta c\delta E[k]}}$.

We use equation (3) from the main text to solve for the degree distribution function:

$$G(t) = Pr(n_i \leq t) = Pr\left(\frac{(1-\delta)}{c_i\delta E[k]}E[1-(1-\delta)^k]wp(1-p) \leq t\right) =$$
$$Pr\left(\frac{(1-\delta)}{t\delta E[k]}E[1-(1-\delta)^k]wp(1-p) \leq c_i\right) = 1 - \frac{(1-\delta)}{t\Delta c\delta E[k]}E[1-(1-\delta)^k]wp(1-p)$$

where we used that the cost distribution is uniform.

Given this we obtain the degree pdf by taking partial derivatives as:

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$$g(t) = \frac{1}{\Delta ct^2}\frac{(1-\delta)}{\delta E[k]}E[1-(1-\delta)^k]wp(1-p)$$

This expression depends on the average degree $E[k]$ and $E[1-(1-\delta)^k]$, so we have to solve for these as follows:

$$E[k] = \int_1^N tg(t)dt = \int_1^N \frac{(1-\delta)}{t\Delta c\delta E[k]}E[1-(1-\delta)^k]wp(1-p)dt =$$
$$\frac{(1-\delta)}{\Delta c\delta E[k]}E[1-(1-\delta)^k]wp(1-p)\int_1^N \frac{1}{t}dt = \frac{(1-\delta)}{\Delta c\delta E[k]}E[1-(1-\delta)^k]wp(1-p)\ln(N)$$

From here we can express:

$$E[k]^2 = \frac{(1-\delta)}{\Delta c\delta}E[1-(1-\delta)^k]wp(1-p)\ln(N) \tag{1}$$

$$E[1-(1-\delta)^k] = \int_1^N (1-(1-\delta)^t)g(t)dt = \int_1^N g(t)dt - \int_1^N (1-\delta)^t g(t)dt =$$
$$1 - \int_1^N \frac{(1-\delta)}{\Delta ct^2\delta E[k]}E[1-(1-\delta)^k]wp(1-p)(1-\delta)^t dt = 1 - \frac{(1-\delta)}{\Delta c\delta E[k]}E[1-(1-\delta)^k]wp(1-p)\int_1^N \frac{(1-\delta)^t}{t^2}dt$$

The last integral can be solved as follows:

$$\int_1^N \frac{(1-\delta)^t}{t^2}dt = \frac{N\ln(1-\delta)(\mathrm{Ei}(N\ln(1-\delta)) - \mathrm{Li}(1-\delta)) - (1-\delta)^N + (1-\delta)N}{N} \equiv A$$

where Ei is the exponential integral function and Li is the logarithmic integral function. We denote this integral by A.

From here we can simplify to:

$$E[1-(1-\delta)^k] = \frac{1}{1 + \frac{p(1-p)(1-\delta)wA}{\Delta c\delta E[k]}} \tag{2}$$

1.2 Proof of Proposition 1

It is easy to show that both n_i^* and n_i^{SO} decreases with c_i. Then we show that $n_i^*(\bar{c}) = 1 < n_i^{SO}(\bar{c})$, which is equivalent to $\bar{c} < \tilde{c}$ where we define $\tilde{c}$ as $n_i^{SO}(\tilde{c}) = 1$.

By using $E[k]$ and $E[1-(1-\delta)^k]$, we can obtain:

$$\frac{wp(1-p)(1-\delta)}{\delta\triangle c}ln(N)\left\{\frac{E[1-(1-\delta)^k]}{E[k]}\right\}^2 + \frac{wp(1-p)(1-\delta)}{\delta\triangle c}A\frac{E[1-(1-\delta)^k]}{E[k]} - 1 = 0 \tag{3}$$

Denote the term $\frac{wp(1-p)(1-\delta)}{\delta\triangle c}$ as ϕ. Then we can compute $\frac{E[1-(1-\delta)^k]}{E[k]}$:

$$\frac{E[1-(1-\delta)^k]}{E[k]} = \frac{-\phi A + \sqrt{\phi^2A^2 + 4\phi ln(N)}}{2\phi ln(N)}$$

Since $\frac{E[1-(1-\delta)^k]}{E[k]} < 1$, we have $-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)} < 2\phi ln(N)$. We know that in the equilibrium, when $c_i = \bar{c}$, $n_i = 1$. Plugging these back into the equilibrium number of neighbours for i (equation 3 in the main text) gives:

$$\bar{c} = \triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{2ln(N)}]$$

Since $\bar{c} > \bar{c} - \underline{c} = \triangle c$, we have $-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)} > 2ln(N)$. Using these two inequalities, we can deduce that $\phi > 1$.

Now we consider the social optimum case, we assume that there is a $\tilde{c}$ which ensures $n^{SO} = 1$. Equation (4) in the main text implies:

$$\tilde{c} = \phi \triangle c[-ln(1-\delta)](1-\delta) \tag{4}$$

Since $\phi > 1$, we have $\bar{c} = \triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{2ln(N)}] < \triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi^2 ln(N)}}{2ln(N)}]$. If we can get $\triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi^2 ln(N)}}{2ln(N)}] < \tilde{c}$, we can indirectly prove that $\bar{c} < \tilde{c}$. $\tilde{c}$ divided by $\triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi^2 ln(N)}}{2ln(N)}]$ gives:

$$\frac{\tilde{c}}{\triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi^2 ln(N)}}{2ln(N)}]} = \frac{[-ln(1-\delta)](1-\delta)}{[\frac{-A + \sqrt{A^2 + 4ln(N)}}{2ln(N)}]}$$

The derivative of the term $[\frac{-A + \sqrt{A^2 + 4ln(N)}}{2ln(N)}]$ with respect to N is:

$$\frac{\partial[\frac{-A + \sqrt{A^2 + 4ln(N)}}{2ln(N)}]}{\partial N} = \frac{2ln(N)\frac{\partial A}{\partial N}[-1 + \frac{A}{\sqrt{A^2 + 4ln(N)}}] + \frac{-4ln(N) - 2A^2 + 2A\sqrt{A^2 + 4ln(N)}}{N\sqrt{A^2 + 4ln(N)}}}{4[ln(N)]^2}$$

According to the expression of A, we have $\frac{\partial A}{\partial N} = \frac{(1-\delta)^N}{N^2} > 0$. Therefore, it is easy to obtain that the sign of $\frac{\partial[\frac{-A + \sqrt{A^2 + 4ln(N)}}{2ln(N)}]}{\partial N}$ is negative. When N large enough, the term $[\frac{-A + \sqrt{A^2 + 4ln(N)}}{2ln(N)}]$ is very small. It is easy to get $\frac{\tilde{c}}{\triangle c[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi^2 ln(N)}}{2ln(N)}]}$ is sufficiently large and $\tilde{c} > \bar{c}$.

This proves that $n_i^*(\bar{c}) = 1 < n_i^{SO}(\bar{c})$. Second, we show that when c is sufficiently small, $n_i^* > n_i^{SO}$. We divide the two FOCs (3) and (4) - see main text:

$$\frac{n_i^*}{n_i^{SO}} = \frac{E[1-(1-\delta)^k]/E[k]}{[-ln(1-\delta)](1-\delta)^{n_i^{SO}}} = \frac{[-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}]/2\phi ln(N)}{[-ln(1-\delta)](1-\delta)^{n_i^{SO}}}$$

Thus it is easy to see that when c_i is very small, n_i^{SO} is very large, and the term $[-ln(1-\delta)](1-\delta)^{n_i^{SO}}$ can be very small. This implies $n_i^{SO} < n_i^*$ when c is sufficiently small. Since the order of n_i and n_i^{SO} changes between very small and large c, there has to be a c_i^* for which $n_i = n_i^{SO}$.

1.3 Proof of Proposition 2

Non-monotonicity of E[k] First, we prove the non-monotonic pattern of $E[k]$ with respect to p. By using $E[k]$ and $E[1-(1-\delta)^k]$, we can obtain:

$$E^2[k] + \frac{wp(1-p)(1-\delta)A}{\delta \triangle c} E[k] - \frac{wp(1-p)(1-\delta)}{\delta \triangle c} ln(N) = 0 \tag{5}$$

The derivative with respect to p is:

$$2E[k]\frac{\partial E[k]}{\partial p} + \frac{wp(1-p)(1-\delta)A}{\delta \triangle c}\frac{\partial E[k]}{\partial p} = \frac{w(1-\delta)}{\delta \triangle c}(ln(N) - AE[k])(1-2p) \tag{6}$$

Through equation (5), we know that $E^2[k] = \frac{wp(1-p)(1-\delta)}{\delta \triangle c}(ln(N) - AE[k]) > 0$, thus when $0 < p < \frac{1}{2}$, we have $\frac{\partial E[k]}{\partial p} > 0$ and when $\frac{1}{2} < p < 1$, we have $\frac{\partial E[k]}{\partial p} < 0$.

Non-monotonicity of the utility difference We have known that the utility of worker i is:

$$U_i = pw + \frac{1}{2}\frac{1}{c_i}[\frac{1-\delta}{\delta}\frac{E[1-(1-\delta)^k]}{E[k]} wp(1-p)]^2 \tag{7}$$

The difference of utility is:

$$U_i - U_j = \frac{1}{2}(\frac{1}{c_i} - \frac{1}{c_j})[\frac{1-\delta}{\delta}\frac{E[1-(1-\delta)^k]}{E[k]} wp(1-p)]^2 \tag{8}$$

Since we know that $\frac{E[1-(1-\delta)^k]}{E[k]} = \frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{2\phi ln(N)} = \frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{\frac{2wp(1-p)(1-\delta)}{\delta \triangle c} ln(N)}$. So the difference of utility is:

$$U_i - U_j = \frac{1}{2}(\frac{1}{c_i} - \frac{1}{c_j})[\frac{1-\delta}{\delta}\frac{E[1-(1-\delta)^k]}{E[k]} wp(1-p)]^2 = \frac{1}{2}(\frac{1}{c_i} - \frac{1}{c_j})[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{2ln(N)}\triangle c]^2 \tag{9}$$

The derivative of it with respect to p is:

$$\frac{\partial(U_i - U_j)}{\partial p} = (\frac{1}{c_i} - \frac{1}{c_j})[\frac{-\phi A + \sqrt{\phi^2 A^2 + 4\phi ln(N)}}{2ln(N)}\triangle c]\frac{\triangle c}{2ln(N)}[-A + \frac{\phi A^2 + 2ln(N)}{\sqrt{\phi^2 A^2 + 4\phi ln(N)}}]\frac{w(1-\delta)}{\delta}(1-2p) \tag{10}$$

We have $-A + \frac{\phi A^2 + 2ln(N)}{\sqrt{\phi^2 A^2 + 4\phi ln(N)}} > 0$ because $[\phi A^2 + 2ln(N)]^2 > A^2[\phi^2 A^2 + 4\phi ln(N)]$. Therefore, the sign $(1-2p)$ determines the sign of $\frac{\partial(U_i - U_j)}{\partial p}$. We can conclude that when $0 < p < \frac{1}{2}$, the difference of utility increases with p, and when $\frac{1}{2} < p < 1$, the difference of utility decreases with p.