

Exposure to the New Wave of Offshoring: An Unemployment Perspective *

Rui Zhang[†]

International School of Economics and Management, Capital University of
Economics and Business, 100070 Beijing, China

Abstract

This paper examines the impact of increasing service offshoring in a two-sector economy. We find that it leads to lower domestic unemployment if the marginal task-specific offshoring cost in the service sector is sufficiently large. Under this condition, the jobs created due to enhanced productivity outweigh the jobs that are destroyed. The reduction in unemployment increases the cost of hiring domestic workers, thus encouraging firms in the manufacturing sector to increase their offshoring scale and productivity. This, in turn, increases their cost savings and may lead to a further decrease in unemployment. Hence, complementarity between two sectors' offshoring activities may emerge. We calibrate the model using economic parameters from Belgium, and the calibration results predict varied unemployment trends and impacts on manufacturing sector offshoring activities with different task-specific offshoring cost schedules.

Keywords: Offshoring, Unemployment, Complementarity.

JEL classification: F1, F11, F16, F23

*Rui Zhang. I would like to thank Martí Mestieri, Yann Bramoullé, Thomas Chaney, Miaomiao Dong, Patrick Fève, Guido Friebel, Jieying Hong, Giammario Impullitti, Matthias Kredler, Shiyuan Pan, Franck Portier, Fabien Postel-Vinay, Gilles Saint-Paul, Konstantinos Tatsiramos and Robert Ulbricht for their helpful advice. I am also thankful to the two anonymous referees who provided useful comments that significantly improved the paper. Finally I would like to thank Research and Innovation Centre of Metropolis Economic and Social Development (CUEB) and NSFC No.71601131 for their financial supports. All errors are mine.

[†]E-mail: ruizhangecon@163.com, International School of Economics and Management, CUEB

1 Introduction

Over the past decades, offshoring developed in two main waves.¹ The first wave was characterized by material offshoring such as consumer electronics, textiles and apparel, and footwear and leather goods.² Production is simply relocated from high-wage to low-wage countries. The second wave was characterized by service relocation, the feasibility of which was facilitated by communication technology improvements. In fact, as the offshoring wave reached offices and white-collar jobs, “tasks that were previously viewed as non-traded became freely traded. The classic example is the moving of US call centers to India” (Baldwin, 2006, p.23). The OECD (2007 b) reports that service sector outsourcing has showed strong growth in OECD countries since 1995.³ Figure 1 shows this trend for selected countries. At the same time, the scale of manufacturing-sector outsourcing in those selected OECD countries continued to grow, despite the much lower growth rate.⁴

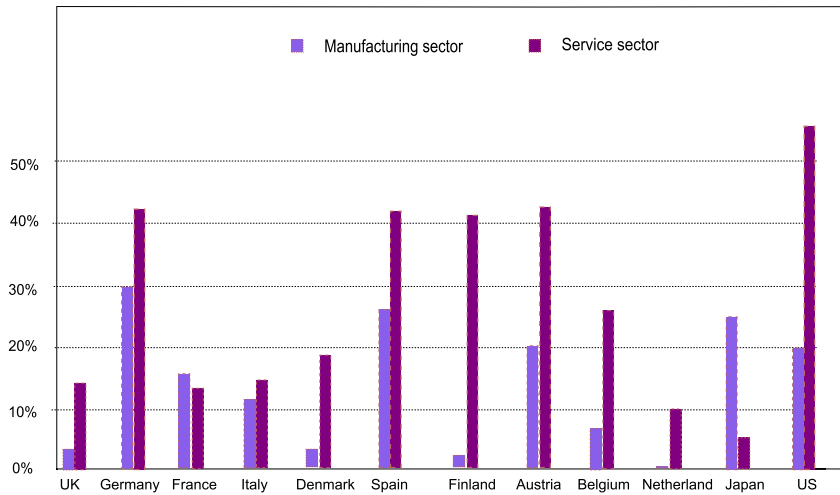


Figure 1: Growth of outsourcing abroad in selected OECD countries from 1995 to 2005

Source: OECD, STAN and LFS database.

Economists and politicians agree that the home country can benefit from offshoring activities by taking advantage of low-cost resources. However, concerns have also been raised regarding its impact on domestic job security. The issue of whether increasing service relocation will destroy domestic jobs has

¹See Bottini, Ernst and Luebker (2007) for a review of trends in offshoring.

²See Alic and Harris (1986), Magaziner and Patinkin (1989), Yoffie and Cassares (1994), Waldinger (1986) and Gereffi (1993) for introductions to material offshoring activities.

³UNCTAD found that 39 percent of the top 500 European firms got involved in the offshoring of services (UNCTAD 2004).

⁴In terms of the level of offshoring extent, service offshoring is also large though it is generally lower than that in the manufacturing sector, especially in some European countries. For example, in Belgium, the offshoring extent is 0.21 in the service sector and the offshoring extent is 0.38 in the manufacturing sector. Therefore, the growth in service offshoring is economically significant.

become a contentious political issue. Some estimates predict that the service offshoring boom will lead to a large number of job losses.⁵ However, until now, there has been no empirical evidence supporting this argument.

In this paper, we investigate the impact of increasing service relocation due to improvements in service offshoring technology. We show that its impact on domestic unemployment can be negative, and we determine the factors contributing to this outcome. We also examine how one sector's offshoring activities impact another's through the general equilibrium effect, and we show how complementarity between different sectors' offshoring activities can emerge. To do so, we construct a simple two-sector model and integrate the standard Pissarides (2000) search model of unemployment with the Grossman and Rossi-Hansberg (2008) task trade model. Accordingly, unemployment is due to search frictions, while producing goods requires the completion of a series of tasks.

In Grossman and Rossi-Hansberg's framework, offshoring's impact is separated into three effects. The first is the productivity effect, which results from cost savings. Domestic firms capitalize on cheaper foreign labor, thus reducing their production costs. The productivity effect can be significant when offshoring volumes are positive and the cost schedule for trading tasks rises steeply. This is equivalent to a situation of high marginal task-specific offshoring cost. The second effect is the relative price effect. When foreign workers conduct some offshoring activities, the price of the products is likely to fall, which will push down the real wages of home-country workers. The third effect is the labor replacement effect. When some tasks are moved abroad, demand for domestic workers falls; hence, maintaining full employment requires reductions in worker wages. In this paper, our model allows offshoring activities to occur in both the manufacturing and service sectors.⁶ Therefore, we have an additional reallocation effect that is derived from the intersectoral reallocation of workers across these two sectors. Job losses in sectors experiencing increasing offshoring may require the reallocation of labor from one sector to other sectors that are creating domestic jobs.⁷

We find that increased offshoring scale in the service sector due to technology improvements leads to a lower domestic unemployment rate if the marginal task-specific offshoring cost for trading tasks is sufficiently high. This occurs because higher marginal task-specific offshoring cost implies greater cost savings. If these cost savings are sufficiently large, the resultant influence of the productivity effect will outweigh the negative impacts attributable to the relative price and replacement effects. A decrease in the unemployment rate consequently encourages firms in the manufacturing sector to

⁵Forrester Research predicts that more than 3 million white-collar jobs will be destroyed due to offshoring activities in the United States by 2015, and Deloitte Research predicts that 2 million financial sector jobs will be lost by 2009.

⁶Some previous literature assume that there is one offshorable sector and one non-offshorable sector, like Mitra and Ranjan (2010), Groizard, Ranjan and Rodriguez-Lopez (2014). But here we focus on the manufacturing and service sectors, in each of them, there are some offshorable tasks and non-offshorable tasks.

⁷Many literature make the same assumption that mobility across sectors is perfect to simplify the analysis, like Grossman and Rossi-Hansberg (2008) and Mitra and Ranjan (2010).

increase the scale of their offshoring activities because hiring domestic workers becomes more costly.⁸ This, in turn, raises the cost savings of offshoring for firms in the manufacturing sector (increasing their productivities), thus amplifying the productivity effect of their offshoring activities. Accordingly, more domestic jobs can be created if the marginal task-specific offshoring cost for trading tasks is also sufficiently large in the manufacturing sector. Therefore, complementarity may emerge between these two sectors' offshoring activities. Advances in offshoring technology in either sector can benefit both sectors' offshoring activities. In addition to analyzing the condition that ensures the dominance of the productivity effect—the driving force of job creation, we also shed light on the role of the reallocation effect, which is a new channel described in the present paper. We show that the existence of the reallocation effect across sectors helps to dilute the negative effects of offshoring on domestic employment and makes the situation more likely to occur that an improvement in offshoring technology reduces unemployment.

In addition to providing analytical results, we undertake a calibration exercise using parameters derived from the Belgian economy, which is characterized by a high degree of openness. We calibrate the model with two task-specific offshoring cost schedules, a steep cost schedule versus a flat cost schedule. The calibration results predict that for a steep cost schedule, improvements in offshoring technology in the service sector, starting from current levels, would reduce domestic unemployment and increase the scale of offshoring in the manufacturing sector.⁹ On the contrary, if the task-specific offshoring cost is flat, domestic unemployment would increase, and the scale of offshoring in the manufacturing sector would decrease.

1.1 Literature Review

There is currently much interest in analyzing the effect of globalization on employment. On the theoretical side, Carl Davidson and Steven Matusz (2004) first introduce labor market search frictions into trade models to show how job search efficiency, the job destruction rate and the frequency of job turnover affect comparative advantage. Felbermayr et al. (2011) integrate search unemployment with Meltiz's (2003) firm heterogeneity model to study the effect of reducing trade cost on unemployment and show that decreases in trade costs raise average productivity and reduce domestic unemployment. Helpman and Itskhoki (2010) also investigate the impact of trade liberalization on unemployment using an imperfectly competitive framework with heterogeneous firms. They conclude that increasing trade openness reduces unemployment if relative labor market frictions in the differentiated sector are high. Besides the change in the employment level, the change in the employment trajectory due to the rising trade has also been highly addressed recently, but almost all are empirical research (e.g. Autor, Dorn and Hanson (2013), Utar and Torres-Ruiz (2013) and Utar (2016)). For example, Utar (2016) shows that a Chinese import shock increases the workers' likelihood of moving from the manufacturing sector

⁸In a stylized search model, wages increase with labor market tightness, so they decrease with the unemployment rate.

⁹A steep cost schedule is equivalent to a large marginal task-specific offshoring cost.

to the service sector in Denmark.

Concerning the impact of offshoring activity, several recent theoretical papers analyze the effect of offshoring on demand for domestic labor and are relevant to our analysis. Grossman and Rossi-Hansberg (2008) construct a simple model of trade in tasks and it serves as our basic structure for analysis. But we focus on the employment effects of offshoring instead of wage effects. Mitra and Ranjan (2010) analyze the impact of offshoring on domestic unemployment. They demonstrate that if intersectoral mobility is allowed, when one sector offshores, economy-wide unemployment will decrease and wages will increase because of the productivity effect. The main assumption in their paper is that domestic and foreign workers are complementary. This assumption will simply strengthen the unemployment-reducing effect. Groizard, Ranjan and Rodriguez-Lopez (2014) study the impact of offshoring on unemployment also in a model with labor market frictions and two sectors (one can offshore and one can not). A decline in offshoring costs affects employment through selection effect, productivity effect, and job-reallocation effect. They show that elasticities of substitution across inputs, varieties, and of demand are crucial in determining the employment trends. In contrast to these mentioned literature, this present paper assumes that domestic and foreign workers can be perfectly substituted and that a net replacement effect will emerge and play a negative role in domestic job creation. Moreover, as we allow all sectors to potentially offshore, and the general equilibrium effect of offshoring activity in one sector can potentially become a driving force of job creation.

In addition, there is much empirical research on the impact of offshoring on employment, especially on the impact of service sector offshoring activities. Amiti and Wei (2005) examine the effects of service sector offshoring in the United States and United Kingdom. For the U.S., they find that offshoring is likely to change employment composition but unlikely to change the aggregate level of employment.¹⁰ Liu and Trefler (2011) study how the rise of trade in services with China and India has impacted U.S. labor markets using the US Current Population Surveys. Their estimations suggest small negative effects of services offshoring. Van Welsum and Reif (2006), using data from 14 OECD countries over the 1996-2003 period, show that the net offshoring of services is not associated with a significant decline in employment. Crinò (2010) uses data of nine Western European countries between 1990 and 2004 to undertake empirical analysis, and shows that service offshoring increases the demand for high and medium skilled labor and decreases the demand for low skilled labor.

The above mentioned literature are all about the employment effects of offshoring. Referring to the interactions between sectors' offshoring activities, the relevant literature is relatively rare. The most recent paper by Beverelli, Fiorini and Hoekman (2017) study the effect of increasing service offshoring on manufacturing productivity and show that it is positive. The present paper provides another probable channel to explain its effect on manufacturing sector's productivity. If increasing service offshoring reduces unemployment and increases the cost of hiring domestic workers, it leads a greater extent of

¹⁰See Amiti and Wei (2006). They reach a similar conclusion for the UK and find that offshoring in the service sector has no negative net effect on employment.

offshoring in the manufacturing sector and raises the productivity effect due to the greater cost-savings.

This paper complements the literature in its examination of offshoring's impact on domestic unemployment. It is the first paper to study how one sector's offshoring activities (service offshoring) impact another's (manufacturing offshoring) through the general equilibrium effect or the labor market effect of offshoring. It shows that an increase in offshoring scale in one sector may be due to the general equilibrium effect or the labor market effect of offshoring in other sectors instead of offshoring technology improvement in its own sector. And this may generate the multiplier effect on employment.

The remainder of the paper is organized as follows. Section 2 introduces the basic ingredients of the model. Section 3 solves for the offshoring equilibriums. Section 4 calibrates the model. Finally, section 5 concludes.

2 Basic Closed-Economy General Equilibrium Model

2.1 Set up

In each economy, all consumers have the same lifetime utility function which is given as

$$\int_{t=0}^{\infty} C_t e^{-rt} dt$$

C is the consumption, r is the discount rate (considered identical in each period), and t is the time index. In this economy, the home country produces one consumption good C , which can be seen as a final or composite good, and it is produced using two intermediate goods, X and Y . X is produced in the service sector, while Y is produced in the manufacturing sector.

Assume that the production function is of the following Cobb-Douglas form

$$C = \frac{X^\rho Y^{1-\rho}}{\rho^\rho (1-\rho)^{1-\rho}}$$

ρ captures the weight of input X in producing final good C , and it ranges from 0 to 1. Let p_x and p_y represent the prices of intermediates X and Y , respectively, and let p_c represent the price of final good C . In a perfectly competitive market, the marginal productivity of each intermediate input is equal to its price. Then, we obtain the relative price of intermediate good $\frac{p_y}{p_x} = \frac{1-\rho}{\rho} \frac{X}{Y}$.

Assuming that the final good C is tradable, its price is taken as given and is normalized to 1 for simplicity. Then, the relationship between the price of X and the price of Y can be represented as $p_x^\rho p_y^{1-\rho} = 1$. Thus, we obtain the price of intermediate good Y

$$p_y = \left(\frac{1-\rho}{\rho} \frac{X}{Y} \right)^\rho \quad (1)$$

2.2 Production and Firms' Profit Maximization Problem

The production of intermediate good X involves l_X workers in the service sector according to

$$X = A_1 l_X^\gamma$$

A_1 is the technology or aggregate productivity of producing X in the service sector. $0 < \gamma < 1$ indicates that the marginal output of production decreases as the workforce involved in production increases.¹¹ Similarly, production of intermediate good Y involves l_Y workers according to

$$Y = A_0 l_Y^\gamma$$

We assume that workers can move freely across sectors. Therefore, if we want to investigate unemployment fluctuations, we only need to analyze the optimization problem of one sector, as workers can move from the sector with high unemployment to the sector with low unemployment. This movement eventually equalizes the two sectors' unemployment. In a closed economy, the present discounted value of the firms' profits at time t in the manufacturing sector Y is given by

$$\text{Max}_{l_Y(s), V_Y(s)} \int_t^\infty e^{-r(s-t)} [p_y(s)Y(s) - w_y(s)l_Y(s) - c_Y V_Y(s)] ds$$

$p_y(s)Y(s)$ is the value of production at time s , $w_y(s)l_Y(s)$ is the total wage paid to workers, and $c_Y V_Y$ is the cost of vacancies, where c_Y is the vacancy cost for each vacant job. When a job is unfilled, there is an additive cost to each firm because it has to pay some vacancy costs (for example, the advertising costs). Define $\theta_Y = \frac{V_Y}{U_Y}$ as the appropriate measure of labor market tightness, where V_Y is the number of vacancies posted by firms in the manufacturing sector, and U_Y is the number of unemployed workers searching for jobs in the manufacturing sector. Furthermore, we define a formal and standard constant returns to scale matching function à la Pissarides as $M(V_Y, U_Y) = mV_Y^{1-\phi}U_Y^\phi$. m is the matching quality or matching efficiency, and ϕ is the matching elasticity. Unemployment is due to search frictions.¹² The rate at which vacant jobs are filled is $q(\theta_Y) = \frac{mV_Y^{1-\phi}U_Y^\phi}{V_Y}$, and the rate at which unemployed workers become employed is $\theta_Y q(\theta_Y) = \frac{mV_Y^{1-\phi}U_Y^\phi}{U_Y}$. $q'(\cdot)$ is assumed to be negative by the properties of the matching technology.¹³ Each job is destroyed by an idiosyncratic shock with probability δ in each period. Then, the dynamics of employment in the manufacturing sector is

$$\dot{l}_Y(t) = q(\theta_Y(t))V_Y(t) - \delta l_Y(t)$$

¹¹OECD (1999) estimates that most OECD countries correspond to a range from 0.6 to 0.72.

¹²We can decompose unemployment into different components, such as a cyclical component, for example, job rationing due to market rigidities, and a frictional component, which is due to labor market frictions. Here, labor market friction is assumed to be the only source of unemployment because the Pissarides search model is a very tractable framework for analyzing the unemployment and is widely used.

¹³See Pissarides (2000).

The firms maximize profits subject to the dynamic motion of the employed workers, taking the worker's wage $w_y(s)$ and the price of the intermediate Y , $p_y(s)$, as given. After simplification, we obtain one wage equation in the manufacturing sector

$$w_y = p_y A_0 \gamma l_Y^{\gamma-1} - \frac{(r + \delta)c_Y}{q(\theta_Y)} \quad (2)$$

This is the job creation condition, corresponding to a marginal condition of the demand for labor. The term $p_y \frac{\partial Y}{\partial l_Y}$ is the marginal productivity of an additive worker in the manufacturing sector, and the term $\frac{(r+\delta)c_Y}{q(\theta_Y)}$ captures the expected value of the firms' hiring cost caused by matching friction in the labor market. When hiring a worker, each firm pays the worker's wage, and each firm also pays vacancy cost when the position is unfilled. In our model, this equation determines the equilibrium employment l_Y as a function of w_y , θ_Y and p_y .

2.3 Wage determination and equilibrium condition

A worker's wage is determined by the firm and the worker using the individual Nash bargaining process. The wage of each employed worker is w_y , while each unemployed worker receives b , which can be viewed as the opportunity cost of being unemployed.

The asset value of an employed worker in the manufacturing sector is denoted $V_{E,Y}$, and $V_{U,Y}$ is the asset value of an unemployed worker. The asset values are given as follows

$$rV_{E,Y} = w_y + \delta(V_{U,Y} - V_{E,Y})$$

$$rV_{U,Y} = b + m\theta_Y q(\theta_Y)(V_{E,Y} - V_{U,Y})$$

The benefit of being employed equals the present wage plus the expected loss from shifting from being employed to being unemployed. Similarly, the benefit of being unemployed equals the opportunity cost b plus the expected benefit of being employed, which is given by $m\theta_Y q(\theta_Y)(V_{E,Y} - V_{U,Y})$. By combining the two equations, the surplus gained when an unemployed worker acquires a job can be given by $V_{E,Y} - V_{U,Y} = \frac{w_y - b}{r + \delta + m\theta_Y q(\theta_Y)}$. In addition, the surplus for a firm from an occupied job in the manufacturing sector J_Y equals the shadow value λ . Because wage is determined through the Nash Bargaining process, where the bargaining weights are represented by η and $1 - \eta$ for the worker and the firm, respectively, we obtain the following wage bargaining equation $\frac{\eta}{1-\eta} = \frac{V_{E,Y} - V_{U,Y}}{J_Y}$.¹⁴

Formally, $J_Y = \lambda = \frac{c_Y}{q(\theta_Y)}$ when entry is free. Therefore, we indirectly obtain the difference between an employed and an unemployed worker $V_{E,Y} - V_{U,Y} = \frac{\eta}{1-\eta} \frac{c_Y}{q(\theta_Y)}$; equalizing this with the surplus from being employed, we obtain another wage equation for the manufacturing sector

¹⁴This is derived from the first-order maximization condition of $Max(V_{E,Y} - V_{U,Y})^\eta (J_Y - V)^{1-\eta}$.

$$w_y = b + \frac{\eta c_Y}{1 - \eta} \left(m\theta_Y + \frac{r + \delta}{q(\theta_Y)} \right) \quad (3)$$

This is the labor supply condition. Equation (3) shows that as labor market tightness enters the wage equation through Nash bargaining, increased tightness leads to a higher wage rate. Given labor demand condition (2) and labor supply condition (3), we can characterize our decentralized equilibrium in the manufacturing sector as

$$p_y \frac{\partial Y}{\partial l_Y} = b + \frac{\eta c_Y}{1 - \eta} m\theta_Y + \frac{c_Y}{1 - \eta} \frac{r + \delta}{q(\theta_Y)} \quad (4)$$

Clearly, labor market tightness depends on the marginal productivity of each labor. Higher marginal productivity results in higher labor market tightness.¹⁵

2.4 Occupation Choice and Price Determination

In the steady state, the welfare of unemployed workers should be identical across sectors because mobility across sectors is allowed, and workers can search for jobs in either sector. The labor mobility condition $V_{U,Y} = V_{U,X}$ implies

$$c_Y \theta_Y = c_X \theta_X \quad (5)$$

The market tightness for each sector is inversely proportional to the cost of posting a new job vacancy. For the sake of simplicity, we assume that vacancy cost is identical in both sectors, that is $c_X = c_Y = c$. We then have $\theta_X = \theta_Y = \theta$ where θ is the tightness of the entire economy. Therefore, according to two equilibrium conditions in both sectors and the labor mobility condition, we obtain $p_y \frac{\partial Y}{\partial l_Y} = p_x \frac{\partial X}{\partial l_X}$. That is, when there are no arbitrage opportunities, the marginal productivity of each labor will be identical in both sectors in equilibrium. Therefore, using the two price equations, we obtain the equilibrium of the working population in the manufacturing sector

$$N_Y = \frac{N}{\left(1 + \frac{\rho}{1-\rho}\right)} = N(1 - \rho) \quad (6)$$

The size of the working population of the manufacturing sector increases with the weight of intermediate Y in producing the final good. Moreover, we can compute the price of each intermediate input in the equilibrium, and the price of intermediate Y is

$$p_y = \left(\frac{1 - \rho}{\rho} \right)^{\rho(1-\gamma)} \left(\frac{A_1}{A_0} \right)^\rho \quad (7)$$

¹⁵Similarly, in the service sector, the decentralized equilibrium condition is $p_x \frac{\partial X}{\partial l_X} = b + \frac{\eta c_X}{1 - \eta} m\theta_X + \frac{c_X}{1 - \eta} \frac{r + \delta}{q(\theta_X)}$.

Finally, knowing the price of each intermediate input and the working population in each sector, we can simplify the previous equilibrium conditions as follows

$$\left(\frac{\rho}{1-\rho}\right)^{(1-\gamma)(1-\rho)} \left(\frac{A_1}{A_0}\right)^{\rho-1} A_1 \gamma N^{\gamma-1} \left(1 + \frac{1-\rho}{\rho}\right)^{1-\gamma} = \left(b + \frac{\eta c}{1-\eta} m \theta + \frac{c}{1-\eta} \frac{r+\delta}{q(\theta)}\right) \left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)}\right)^{1-\gamma} \quad (8)$$

Here, there is only one endogenous variable to be determined, that is, the market tightness of the entire economy θ .¹⁶ Indirectly, we can obtain the unemployment rate in the steady state according to $u = \frac{\delta}{\delta + m \theta q(\theta)}$. The unemployment rate is negatively related to labor market tightness.

3 Equilibrium with Offshoring Activity

3.1 Offshoring in One Sector

Here, we first examine the impact of one sector's offshoring activity on the domestic labor market, say, the manufacturing sector. And we assume no offshoring activities in the service sector. We borrow the basic set-up from Grossman and Rossi-Hansberg (2008), where the home country can offshore tasks to countries with lower costs. There is a continuum of Y -tasks that we index by $i \in [0, 1]$. If some tasks in the manufacturing sector are moved to the foreign country, the firm uses the same amount of labor as the firm would use on each task in the home country. That is, domestic and foreign workers can be perfectly substituted. Because Grossman and Rossi-Hansberg (2008) do not specify whether the offshoring is done inside or outside of the domestic firms, as many other papers do, the home technology being transferred abroad can be seen as a multinational relationship in offshoring.

For the sake of simplicity, we assume that the entire measure of tasks required per unit of labor (each worker) can be normalized to 1. Moreover, to complete production, all tasks $i \in [0, 1]$ must be performed. For example, if the firm needs l units of domestic labor input to perform each task i , then l is also the total amount of domestic labor input because $l = l \int_0^1 di$. In this framework, l is endogenous, and the firm can determine optimally how much labor input it will use on each task and indirectly determine how many workers it will hire.

Following Grossman and Rossi-Hansberg (2008), we use $\beta_I t(i) > 1$ to represent the cost of offshoring task i . β_I is a shift parameter that depends on the offshoring technology in the manufacturing sector, and it is indifferent for all tasks that are offshored.¹⁷ $t(i)$ captures the specific cost for a task with rank i , and without loss of generality, the tasks are ranked from 0 to 1. It becomes more difficult to offshore tasks with a higher rank; therefore, the function $t(i)$ is increasing in i and $t'(i) \geq 0$. If $t'(i) = 0$, the offshoring cost is identical for all tasks.

When offshoring is possible and firms decide to move part of their production abroad, they incur the

¹⁶Because the right hand side of the equation (8) is an increasing function of θ , the equilibrium is unique.

¹⁷It depends on the outside trading environment, for example, trade barriers and transportation costs.

cost of hiring foreign workers. To focus our analysis on the home labor market, we assume that there is no search friction in the foreign labor market, and the foreign wage rate w^* is taken as given. Firms in the home country want to perform the tasks $i \in [0, I]$ abroad, and the marginal task I separates tasks $i \in [0, I]$ performed abroad from tasks $i \in [I, 1]$ performed domestically. The scale of offshoring in home country I is determined optimally by the firms. Like Grossman and Rossi-Hansberg, we assume that $t(i) \geq 1$ and $t(0) = 1$.

When firms in the manufacturing sector offshore, we assume that firms need \tilde{l} units of labor input to perform each task.¹⁸ Domestic workers perform tasks in the range $i \in [I, 1]$, and foreign workers perform tasks in the range $i \in [0, I]$; therefore, firms should hire $\tilde{l} \int_I^1 di = \tilde{l}(1 - I) = l_{D,Y}$ domestic workers and $\tilde{l} \int_0^I di = \tilde{l}I = l_{F,Y}$ foreign workers. The wages paid to the home workers in manufacturing sector Y are represented as $w_y \tilde{l} \int_I^1 di = w_y l_{D,Y}$, and the wages paid to foreign workers are represented as $w^* \tilde{l} \int_0^I \beta_I t(i) di = w^* (l_{F,Y} + l_{D,Y}) \int_0^I \beta_I t(i) di$. w_y is the wage rate in the manufacturing sector of the home country, and w^* is the wage rate of the foreign country.

3.1.1 Firms producing Y

When firms use workers from both home and foreign countries, they hire $l_{D,Y}$ domestic workers and $l_{F,Y}$ foreign workers to produce intermediate input Y . $l_{D,Y}$ and $l_{F,Y}$ satisfy the following condition¹⁹

$$I = \frac{l_{F,Y}}{l_{D,Y} + l_{F,Y}} \quad (9)$$

The profit maximization condition for a firm is given as follows

$$\begin{aligned} \underset{l_{D,Y}(s), l_{F,Y}(s), V_Y^D(s), V_Y^F(s)}{\text{Max}} \int_t^\infty e^{-r(s-t)} [p_y(s)Y(s) - cV_Y^D(s) - c^F V_Y^F(s) \\ - w_y(s)l_{D,Y}(s) - w^*(s)(l_{F,Y}(s) + l_{D,Y}(s)) \int_0^I \beta_I t(i) di] ds \end{aligned}$$

V_Y^D is the number of vacancies posted domestically, and V_Y^F is the number of vacancies posted abroad. As previously assumed, there is no search friction in the foreign labor market; the probability of filling a vacancy is 1, and the vacancy cost c^F is 0.

We solve the maximization problem of the firm in the appendix and obtain two equilibrium conditions of which the first is

$$w_y + \frac{(r + \delta)c}{q(\theta_Y)} = p_y \frac{\partial Y}{\partial l_{D,Y}} - w^* \int_0^I \beta_I t(i) di - w^* (l_{F,Y} + l_{D,Y}) \beta_I t(I) \frac{\partial I}{\partial l_{D,Y}} \quad (10)$$

From equation (10), we can see that in equilibrium, the marginal cost of hiring a home worker equals the marginal productivity of the worker plus an extra term due to offshoring activities. The extra term

¹⁸It is not reasonable to believe $l = \tilde{l}$. Due to cost savings, each firm would produce more than in a closed economy, and thus $l < \tilde{l}$.

¹⁹This condition is derived from $(l_{D,Y} + l_{F,Y}) \int_0^I di = l_{F,Y}$.

includes two parts; the first part, $-w^* \int_0^I \beta_I t(i) di < 0$, captures the marginal cost of hiring an additional unit of foreign labor, while the second part, $-w^*(l_{F,Y} + l_{D,Y})\beta_I t(I) \frac{\partial I}{\partial l_{D,Y}} > 0$, captures the marginal benefit from cost reduction because hiring foreign workers is less costly than hiring home workers. Combining these two parts, we obtain the cost savings due to offshoring activities $w^*\beta_I[t(I)I - \int_0^I t(i)di]$, which is positive when $I > 0$. Taking the offshoring technology β_I and foreign wage w^* as given, the cost savings increase with offshoring extent I .

The second equilibrium condition is

$$w^*\beta_I t(I) = w_y + \frac{(r + \delta)c}{q(\theta_Y)} \quad (11)$$

By looking at this equation, we can see that for the marginal task I , there is no difference between hiring a home worker and a foreign worker. Clearly, the equilibrium point for offshoring is determined when the cost of performing the borderline task abroad, which is $w^*\beta_I t(I)$, equals the cost of hiring domestically.

We plug equation (11) back into equation (10) to obtain the job creation condition.²⁰ It gives us the following

$$p_y \frac{\partial Y}{\partial l_{D,Y}} F[I] = w_y + \frac{(r + \delta)c}{q(\theta_Y)} \quad (12)$$

Then, we have two equations, (11) and (12), jointly determining two endogenous variables, labor market tightness in the manufacturing sector and the extent of offshoring.²¹ $F(I) = \frac{t(I)}{\int_0^I t(i)di + t(I)(1-I)}$. It captures the productivity effect, which is greater than 1 when the task-specific offshoring cost increases with the rank of task i , $t'(i) > 0$.²² Moreover, the productivity effect increases with the extent of offshoring, $\frac{\partial F(I)}{\partial I} > 0$, if $t'(i) > 0$.²³

Lemma 1. *The productivity effect increases with the extent of offshoring I if the task-specific offshoring cost increases with the rank of task i , $t'(i) > 0$.*

When the task-specific offshoring cost becomes steeper, for each level of the offshoring extent I , $t'(I)$ becomes larger and indirectly leads to a greater marginal productivity effect $F'(I)$. As shown in Grossman and Rossi-Hansberg (2008), around $I = 0$, the marginal productivity effect is close to 0; therefore, the enhanced productivity is negligible. In the case that the task-specific offshoring cost, $t(i)$, is identical for all tasks belonging to $[0, 1]$ which implies that $t'(i) = 0$, the cost savings equal 0, and the productivity effect disappears.

²⁰Because domestic and foreign workers can be perfectly substituted in terms of productivity, we have $p_y \frac{\partial Y}{\partial l_{D,Y}} = p_y \frac{\partial Y}{\partial l_{F,Y}}$.

²¹We can obtain the wage from the labor supply side, which is identical to that of the closed economy and is determined by individual Nash bargaining.

²²When the task-specific offshoring cost function $t(i)$ increases with the rank of task i , we have $t(I)I > \int_0^I t(i)di$.

²³This is because $\frac{\partial F(I)}{\partial I} = \frac{t'(I) \int_0^I t(i)di}{[\int_0^I t(i)di + t(I)(1-I)]^2} > 0$.

3.1.2 Worker Reallocation

In equilibrium, the marginal productivity of each worker is identical across sectors given that the vacancy cost is identical across sectors and that workers can move freely from one sector to another. So we have

$$p_y \frac{\partial Y}{\partial l_{D,Y}} F(I) = p_x \frac{\partial X}{\partial l_{D,X}}$$

After simplification, we have the relative supply of the working population in the manufacturing sector

$$\frac{N_Y}{N_X} = \frac{1 - \rho}{\rho} (1 - I) F(I) \quad (13)$$

Proposition 1. *When only one sector (manufacturing) engages in offshoring activities, more workers, compared with the closed economy, choose to work in the sector (service) without offshoring activities.*

Proof. See appendix. □

This result holds regardless of how economy-wide and sectorial unemployment evolve. Offshoring activity in the manufacturing sector has two effects on worker allocation between sectors. On the one hand, offshoring enhances the productivity of firms and pushes up workers' wages in the manufacturing sector. This attracts some workers from the service sector without offshoring activities. On the other hand, offshoring relocates some jobs formerly performed by domestic workers abroad and drives down workers' wages. This encourages some workers to flow into the service sector. With our set up, the latter effect dominates the former. Therefore, there is a net flow of workers from the manufacturing sector with offshoring activities toward the service sector without. Thus, the number of workers in the manufacturing sector is

$$N_Y = \frac{N}{1 + \frac{\rho}{1-\rho} \frac{1}{F(I)(1-I)}} \quad (14)$$

3.1.3 Prices of Intermediate Goods

The price of intermediate input Y which can be offshored becomes

$$p_y = \left(\frac{1 - \rho}{\rho} \right)^{\rho(1-\gamma)} \left(\frac{A_1}{A_0} \right)^\rho \left(\frac{1}{F(I)} \right)^{\gamma\rho} \quad (15)$$

Compared to the closed economy, the price of input Y decreases with offshoring scale I because $F(I) > 1$. Firms in the manufacturing sector produce more due to enhanced productivity, which drives down the price of intermediate input Y . On the contrary, the price of input X , which is not offshored, increases because the relative supply for this intermediate input is lower.

Proposition 2. *Offshoring reduces the price of the intermediate input which is offshored, and raises the price of the intermediate input which is not offshored.*

3.1.4 Equilibrium with Offshoring

After determining the equilibrium price of intermediate input Y in equation (15), the size of the working population in the manufacturing sector in equation (14) and given the wage level of the workers in the equation (3), we can re-characterize the equilibrium condition (12) as follows

$$\begin{aligned} \left(\frac{1-\rho}{\rho}\right)^{\rho(1-\gamma)} \left(\frac{A_1}{A_0}\right)^\rho \underbrace{\left(\frac{1}{F(I)}\right)^{\gamma\rho}}_{\text{relative-price}} \gamma A_0 N^{\gamma-1} \left[1 + \frac{\rho}{1-\rho} \overbrace{\frac{1}{F(I)(1-I)}}^{\text{reallocation}}\right]^{1-\gamma} \underbrace{(1-I)^{1-\gamma}}_{\text{replacement}} \overbrace{F(I)}^{\text{productivity}} \\ = \left(b + \frac{\eta c}{1-\eta} m\theta + \frac{1}{1-\eta} \frac{(r+\delta)c}{q(\theta)}\right) \left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)}\right)^{1-\gamma} \end{aligned} \quad (16)$$

Here, we specify each individual effect due to offshoring activities. They include relative price, reallocation, replacement and productivity effects. The productivity and reallocation effects are positive, while the relative price and replacement effects are negative.²⁴ The product of those four individual effects, which is referred to as the aggregate effect, is denoted by $\Upsilon(I)$. It can be simplified into

$$\Upsilon(I) = [F(I)^{\frac{1-\gamma\rho}{1-\gamma}} (1-I) + \frac{\rho}{1-\rho} F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}}]^{1-\gamma}$$

The larger the aggregate effect, the higher the market tightness and the lower the unemployment rate.²⁵

Proposition 3. *When offshoring technology improves in the manufacturing sector (β_I decreases), if the aggregate effect $\Upsilon(I)$ increases, then unemployment in the home country falls.*

If all tasks have identical task-specific offshoring costs, that is, $t'(i) = 0$ for $i \in [0, 1]$, then the cost savings are null. In this case, the productivity effect disappears and an increase in the offshoring scale due to improved offshoring technology simply raises the unemployment rate of the home country.

²⁴Given that workers can move freely across sectors, in equilibrium, labor market tightness and unemployment are identical in both sectors, so we use θ instead of θ_Y to represent the tightness of the entire market.

²⁵We also have another condition (11) that jointly determines the equilibrium with job creation condition. It shows how the cost of hiring domestic workers affects the offshoring extent. From this condition, we know that improvement in offshoring technology encourages firms to offshore a greater extent of tasks i because offshoring becomes cheaper, which will in turn affect the aggregate effect of condition (12). Grossman and Rossi-Hansberg (2008) also conclude that offshoring scale increases with offshoring technology. That is, when β_I decreases, I increases in equilibrium. Therefore, the impact of improvement in offshoring technology on the aggregate effect is equivalent to that of increase in the extent of offshoring. When we study the impact of improving offshoring technology on the labor market tightness or the unemployment rate analytically, we can simply focus on how the change in the scale of offshoring affects the labor market tightness in the partial equilibrium.

If the task-specific offshoring cost differs in tasks, we have $t'(i) > 0$. Then, the derivation of the aggregate effect with respect to the offshoring extent gives

$$\frac{\partial [F(I)^{\frac{1-\gamma\rho}{1-\gamma}}(1-I) + \frac{\rho}{1-\rho}F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}}]^{1-\gamma}}{\partial I} = [F(I)^{\frac{1-\gamma\rho}{1-\gamma}}(1-I) + \frac{\rho}{1-\rho}F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}}]^{-\gamma} \left[\frac{1-\gamma\rho}{1-\gamma}F'(I)F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}}(1-I) + \frac{\rho\gamma}{1-\gamma}F'(I)F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}-1} - F(I)^{\frac{1-\gamma\rho}{1-\gamma}} \right]$$

As $\frac{\partial F(I)}{\partial I} = \frac{t'(I) \int_0^I t(i) di}{[\int_0^I t(i) di + t(I)(1-I)]^2}$, when the offshoring extent I is close to 0, $F'(I)$ is close to 0, and the derivation of the aggregate effect is close to $-(1-\rho)^\gamma$. On this condition, the aggregate effect decreases with offshoring extent I . However, as argued in Grossman and Rossi-Hansberg (2008), $F'(I)$ can be large given two conditions: first, that there is a positive offshoring scale $I > 0$; second, that the cost schedule for trading tasks rises steeply. Equivalently, it implies that if the marginal task-specific offshoring cost for trading tasks $t'(I)$ is sufficiently large, the aggregate effect can increase with the offshoring extent I .²⁶

We illustrate the role of $t'(I)$ in determining cost savings in Figure 2. As shown in equation (10), the term $w^*\beta_I[t(I)I - \int_0^I t(i)di]$ captures the cost savings when the home firms offshore tasks with rank $i \in [0, I]$ abroad. Here we try two cost schedules, relatively steep vs. relatively flat. For each β_I , we have different equilibrium extent of offshoring; a relatively steep task-specific cost function implies a smaller offshoring scale in equilibrium. In the figure, $t_2(I)$ represents the relatively steep cost schedule. Its cost savings $w^*\beta_I[t_2(I_2)I_2 - \int_0^{I_2} t_2(i)di]$ are represented by $B + C$. Similarly, the cost savings for the relatively flat cost schedule are represented by $A + C$. Therefore, $B - A$ captures the extra benefits from the higher degree of steepness. If this difference is sufficiently large, the extra benefit from a higher degree of steepness can be large enough to ensure that the positive effects of offshoring outweigh the negative effects and thus create jobs.²⁷

²⁶Grossman and Rossi-Hansberg (2008) consider full employment and argue, “The first bit of offshoring drives down the wages of domestic workers. This is because the fact that the productivity effect rests on the cost savings for inframarginal tasks, and there are no such tasks when the complete production process is performed initially at home.”

²⁷In Grossman and Rossi-Hansberg (2008), the effects of offshoring are reflected by the change in wages. However, in our model with search frictions, the effects of offshoring are reflected by both the change in wages and change in unemployment rate.

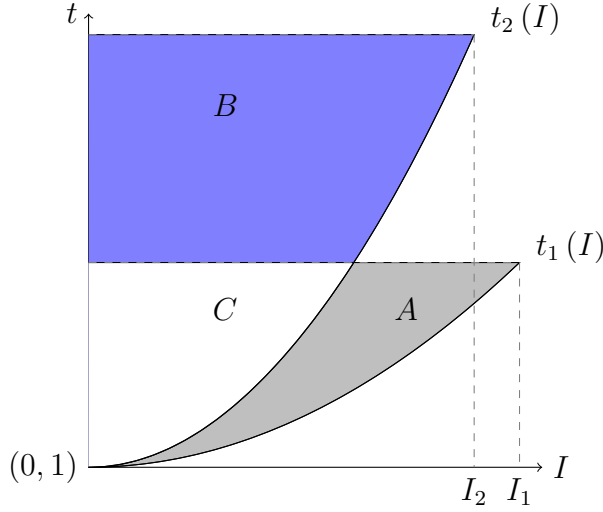


Figure 2: Cost savings for different offshoring cost functions

3.1.5 The Role of Reallocation Effect

In contrast to the work of Grossman and Rossi-Hansberg, we have an additional reallocation effect that can impact unemployment in the two-sector model. Without this effect, the aggregate effect is the combination of the productivity effect, the price effect and the replacement effect. It can be simplified into $F(I)^{1-\gamma\rho}(1-I)^{1-\gamma}$, and its derivation is

$$\frac{\partial F(I)^{1-\gamma\rho}(1-I)^{1-\gamma}}{\partial I} = F(I)^{-\gamma\rho}(1-I)^{-\gamma}[(1-\gamma\rho)(1-I)F'(I) - (1-\gamma)F(I)]$$

Clearly, when $F'(I) > \frac{(1-\gamma)F(I)}{(1-\gamma\rho)(1-I)}$, the derivation is positive and the combination of the three effects increases with the offshoring scale. When we take into account the reallocation effect, the derivation of the aggregate effect with respect to the offshoring scale is positive when $F'(I) > \frac{(1-\gamma)F(I)}{(1-\gamma\rho)(1-I)+\gamma\rho F(I)^{-1}}$. The term $F(I)^{-1}$ is clearly positive. Therefore, the reallocation effect across sectors simply makes it more likely that an improvement in offshoring technology reduces unemployment.

Corollary 1.

The existence of the reallocation effect across two sectors makes it more likely that an improvement in offshoring technology reduces unemployment.

When the manufacturing sector offshores, the decrease in demand for domestic labor at the intensive margin exerts downward pressure on the workers wages in the manufacturing sector, leading to a net outflow of workers from the manufacturing sector to the service sector, in which the demand for domestic workers becomes higher due to the offshoring activities in the manufacturing sector.²⁸ And

²⁸The decrease in demand for domestic labor at the intensive margin means that the proportion of domestic workers demanded by firms to perform each task becomes lower.

the reallocation of workers across sectors helps to dilute the negative effects of manufacturing sector's offshoring on the economy-wide employment.²⁹

3.2 Offshoring in Two Sectors

Here, we focus on two sectors' offshoring activities to analyze the impact of progress in offshoring technology in the service sector. We assume that firms in the service sector offshore tasks with index j . The relative supply of workers becomes

$$\frac{N_Y}{N_X} = \frac{1 - \rho}{\rho} \frac{(1 - I)F(I)}{(1 - J)F(J)} \quad (17)$$

As previously stated, to capture the change in the unemployment rate of the home country, we need to analyze one sector's activity because mobility across sectors is allowed. Therefore, the price of the intermediate input Y becomes

$$p_y = \left(\frac{1 - \rho}{\rho} \right)^{\rho(1-\gamma)} \left(\frac{A_1}{A_0} \right)^\rho \left(\frac{F(J)}{F(I)} \right)^{\gamma\rho} \quad (18)$$

Clearly, the price of intermediate input Y depends on the relative productivity effect $\frac{F(J)}{F(I)}$ because it can capture the change in the relative production which indirectly affects the relative price. For simplification, we assume that firms in both sectors offshore tasks to the countries with the same wage rate w^* . β_I represents the non task-specific offshoring cost in the manufacturing sector, and β_J represents the non-task-specific offshoring cost in the service sector. At the cutoff, the firm will be indifferent between hiring domestic and foreign workers; therefore, we have two equations as follows

$$w^* \beta_I t(I) = b + \frac{\eta c}{1 - \eta} m \theta_Y + \frac{1}{1 - \eta} \frac{(r + \delta)c}{q(\theta_Y)} \quad (19)$$

$$w^* \beta_J t(J) = b + \frac{\eta c}{1 - \eta} m \theta_X + \frac{1}{1 - \eta} \frac{(r + \delta)c}{q(\theta_X)} \quad (20)$$

Because workers can move freely across sectors, we have $\theta_X = \theta_Y = \theta$. Combining equations (19) and (20), we obtain the relationship between marginal task I and marginal task J in equilibrium³⁰

$$\beta_I t(I) = \beta_J t(J) \quad (21)$$

²⁹The reallocation effect helps to dilute the negative effects but cannot outweigh the negative effects. Without the productivity effect, the combination of the other three effects is $[(1 - I) + \frac{\rho}{1 - \rho} \frac{1}{F(I)}]^{1-\gamma} (\frac{1}{F(I)})^{\gamma\rho}$ and it is lower than that in the closed economy.

³⁰In the case of symmetric offshoring, the non-task-specific offshoring costs are the same for two sectors and the offshoring cost functions have the same form; thus, firms in both sectors offshore to the same extent abroad in equilibrium, i.e., $I = J$.

In addition to equations (19) and (20), another equilibrium condition provides the relationship between market tightness and offshoring extent

$$\begin{aligned} \left(\frac{1-\rho}{\rho}\right)^{\rho(1-\gamma)} \left(\frac{A_1}{A_0}\right)^\rho \underbrace{\left(\frac{F(J)}{F(I)}\right)^{\gamma\rho}}_{\text{relative-price}} \gamma A_0 N^{\gamma-1} \left[1 + \frac{\rho}{1-\rho} \overbrace{\frac{1}{F(I)(1-I)}}^{\text{reallocation}}\right]^{1-\gamma} \underbrace{(1-I)^{1-\gamma}}_{\text{replacement}} \overbrace{F(I)}^{\text{productivity}} \\ = \left(b + \frac{\eta c}{1-\eta} m\theta + \frac{1}{1-\eta} \frac{(r+\delta)c}{q(\theta)}\right) \left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)}\right)^{1-\gamma} \end{aligned} \quad (22)$$

Thus, we have three endogenous variables: offshoring extent in each sector, I and J , and labor market tightness θ . We can obtain them using equations (19), (20) and (22).

The product of those individual effects is the aggregate effect. We denote the aggregate effect of offshoring as $\Upsilon(I, J)$. When both sectors engage in offshoring activities, it can be simplified into

$$\Upsilon(I, J) = F(J)^{\gamma\rho} [F(I)^{\frac{1-\gamma\rho}{1-\gamma}} (1-I) + \frac{\rho}{1-\rho} F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}} F(J)(1-J)]^{1-\gamma}$$

When offshoring technology improves in the service sector, which means that β_J decreases, firms in the service sector increase their offshoring scale J . If we consider its impact on the unemployment rate, offshoring activities and productivity in the manufacturing sector, we can draw the following conclusion.

Proposition 4. *When offshoring technology improves in the service sector (i.e., β_J decreases), if the aggregate effect $\Upsilon(I, J)$ increases:*

- i) *The unemployment rate in the home country falls;*
- ii) *Each firm in the manufacturing sector enlarges its offshoring scale, I , and productivity in the manufacturing sector increases.*

If the improvement in service offshoring technology reduces the unemployment rate, firms in the manufacturing sector expands their offshoring scale because hiring domestic workers becomes more costly, this improving the productivity of firms in the manufacturing due to the enhancing cost-savings.

The derivation of $\Upsilon(I, J)$ with respect to I gives

$$\begin{aligned} \frac{\partial \Upsilon(I, J)}{\partial I} = \Lambda \left[\frac{1-\gamma\rho}{1-\gamma} F(J)^{\gamma\rho} F(I)^{\frac{1-\gamma\rho}{1-\gamma}-1} F'(I)(1-I) - F(J)^{\gamma\rho} F(I)^{\frac{1-\gamma\rho}{1-\gamma}} \right. \\ \left. + \frac{\rho}{1-\rho} \frac{\gamma(1-\rho)}{1-\gamma} F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}-1} F'(I) F(J)^{1+\gamma\rho} (1-J) \right] \end{aligned}$$

Where $\Lambda = (1-\gamma)[F(I)^{\frac{1-\gamma\rho}{1-\gamma}} (1-I) + \frac{\rho}{1-\rho} F(I)^{\frac{\gamma(1-\rho)}{1-\gamma}} F(J)(1-J)]^{-\gamma}$ and it is positive. Additionally,

the derivation of $\Upsilon(I, J)$ with respect to J gives

$$\begin{aligned} \frac{\partial \Upsilon(I, J)}{\partial J} = & A \left[\frac{\gamma \rho}{1 - \gamma} F(J)^{\gamma \rho - 1} F(I)^{\frac{1 - \gamma \rho}{1 - \gamma}} F'(J)(1 - I) - \frac{\rho}{1 - \rho} F(I)^{\frac{\gamma(1 - \rho)}{1 - \gamma}} F(J)^{\gamma \rho + 1} \right. \\ & \left. + \frac{\rho}{1 - \rho} \frac{1 - \gamma + \gamma \rho}{1 - \gamma} F(I)^{\frac{\gamma(1 - \rho)}{1 - \gamma}} F'(J) F(J)^{\gamma \rho} (1 - J) \right] \end{aligned}$$

As in our previous analysis, if the marginal task-specific offshoring cost $t'(J)$ is small for task j , $F'(J)$ is small, and $\frac{\partial \Upsilon(I, J)}{\partial J}$ is negative. In this case, increasing the offshoring extent J in the service sector reduces the aggregate effect and raises the domestic unemployment rate. On the contrary, if the marginal task-specific offshoring cost $t'(J)$ is sufficiently large for task j and $\frac{\partial \Upsilon(I, J)}{\partial J}$ is positive, then increasing the offshoring extent J in the service sector increases the aggregate effect and reduces the domestic unemployment. Consequently, the progress of offshoring technology in the service sector, which leads to greater offshoring extent J , can push the aggregate effect in either direction and can thereby either increase or decrease domestic unemployment. When we compare a situation in which services can not be offshored with a situation in which they can be offshored, the results are thus ambiguous.

Referring to the reallocation effect when service offshoring is allowed, if $F'(I) > \frac{(1 - \gamma)F(I)}{(1 - \gamma \rho)(1 - I)}$, the derivation is positive and the combination of the three effects increases with the offshoring scale. When we take into account the reallocation effect, the derivation of the aggregate effect with respect to the offshoring scale is positive when $F'(I) > \frac{(1 - \gamma)F(I)}{(1 - \gamma \rho)(1 - I) + \gamma \rho F(I)^{-1} F(J)(1 - J)}$. The term $F(I)^{-1} F(J)(1 - J)$ is clearly positive. Therefore, the openness to offshoring activities in the service sector does not change the conclusion. The reallocation effect across sectors dilutes the negative effects of offshoring on domestic employment.

3.3 General Equilibrium Effect

Now, we look at the general equilibrium effect of a change in the service sector's offshoring technology on the manufacturing sector's offshoring activity. First, we consider the condition for which the marginal task-specific offshoring cost $t'(I)$ is sufficiently large for task i in the initial equilibrium of the manufacturing sector. Under this condition, if $t'(J)$ is sufficiently large for task j as well in the initial equilibrium, the progress of offshoring technology in the service sector will increase the offshoring scale in the manufacturing sector via increasing the cost of hiring domestic workers. That is, progress of offshoring technology in the service sector increases the aggregate effect and reduces the domestic unemployment rate. This makes hiring home workers more expensive, and it makes the firms in the manufacturing sector increase the extent of their offshoring activities, leading to an improvement in their productivities. This general equilibrium effect will reinforce the dominance of the productivity effect in the manufacturing sector and in turn lead to a further decrease in domestic unemployment.³¹

³¹Mathematically, when both $t'(I)$ and $t'(J)$ are sufficiently large in the initial equilibrium, the progress of offshoring technology in the service sector raises the aggregate effect in equation (22) and increases labor market tightness (reduces

On the contrary, if $t'(J)$ is small for task j in the initial equilibrium, the progress of offshoring technology in the service sector will decrease the offshoring volume in the manufacturing sector to a smaller extent via reducing the cost of hiring domestic workers, leading to a decrease in the productivity of the firms in the manufacturing sector. Moreover, this general equilibrium effect will weaken the dominance of the productivity effect in the manufacturing sector and lead to a further increase in domestic unemployment. In both cases, the general equilibrium effect will amplify the impact of offshoring technology progress in the service sector, either positively or negatively.

Second, we consider the condition where, for the manufacturing sector, in the initial equilibrium, the marginal task-specific offshoring cost $t'(I)$ is not sufficiently large. Under this condition, the general equilibrium effect will weaken the impact of offshoring technology progress in the service sector on the domestic unemployment rate.

Figure 3 shows the role of the general equilibrium effect in different conditions. The solid arrow represents the impact of offshoring technology progress in the service sector on domestic labor market tightness (the higher labor market tightness, the lower the unemployment rate), and the dotted arrow represents the general equilibrium effect, stemming from the change in the extent of offshoring of the manufacturing sector, on labor market tightness.

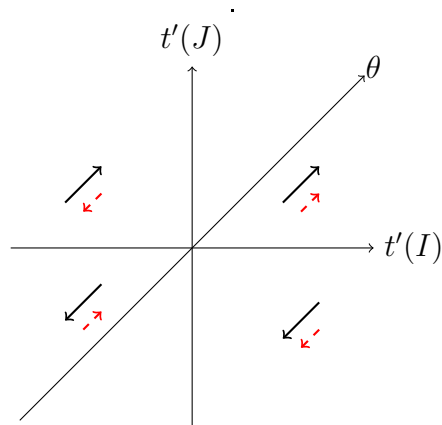


Figure 3: When offshoring technology improves in the service sector, $t'(J)$ increases

domestic unemployment). The feedback effect on the manufacturing sector can be obtained from equation (20), from which we can conclude that greater labor market tightness encourages firms to increase offshoring scale in the manufacturing sector because hiring domestic workers becomes more costly. This increased scale of offshoring in turn reinforces the strength of the productivity effect in the manufacturing sector and leads to a further increase in the aggregate effect in equation (22), resulting in lower unemployment rate.

4 Numerical Illustration

4.1 Calibration for Belgium

We undertake a calibration exercise using economic parameters for Belgium. I choose Belgium as the target country for two reasons. First, the service offshoring scale is large. It is larger than other European countries, such as the UK and Germany, and much larger than the US. Second, the service offshoring scale increases very fast, for example, between 1995 and 2000, the annual growth rate of service offshoring reached 7%. At the same time, the manufacturing sectors offshoring activities grow very slightly. Therefore, it is quite clear to see the net effect of increasing service relocation on unemployment without considering the impact of manufacturing sectors offshoring activities.³² We discuss the choice of some crucial parameters below.³³

Here, we use the month as the unit of time. The monthly rate of job destruction δ is set to 0.013 and the discount rate r is set to 0.008 based on the parameter values used by Van der Linden et al. (2001). For the elasticity of matching function ϕ for Belgium, Petrongolo and Pissarides (2000) estimate it to be 0.6. Therefore, the job finding rate can be given as $m\theta^{0.4}$. In Van der Linden et al. (2001), the estimate of labor market tightness in Belgium is 0.14, and hence, the scale parameter m can be obtained indirectly using the job destruction rate δ and the unemployment rate u according to $u = \frac{\delta}{\delta + m\theta^{0.4}}$. Then, we obtain an estimate of $m = 0.29$. As for the worker's share of surplus, we choose $\eta = 0.77$, which is from Van der Linden et al. (2001) as well. According to the OECD report, the net replacement ratio $\frac{b}{w}$ for Belgium is approximately 0.35.

Next, we set the parameters related to production activity. Knowing the consumption of each intermediate input, which can be found in the OECD STAN input-output tables, we can estimate the weight parameter ρ according to $\frac{p_y Y}{p_x X} = \frac{1-\rho}{\rho}$, and the estimate is $\rho = 0.476$. We estimate the production function parameter and the technology of production in each sector according to the following equations, which are normally used to obtain the Solow residual

$$\ln Y = \ln A_0 + \gamma_Y \ln l_Y + (1 - \gamma_Y) \ln K_Y + \epsilon_Y$$

and

$$\ln X = \ln A_1 + \gamma_X \ln l_X + (1 - \gamma_X) \ln K_X + \epsilon_X$$

The estimates of the production parameters for two sectors yield $\gamma_Y = 0.59$ and $\gamma_X = 0.77$. These results are reasonable given that the estimate of the aggregate production parameter is 0.64 for Belgium.³⁴

³²It is also shown in Bernhard (2008) that offshoring in Belgium is on the rise and that this rise is mainly driven by services.

³³We choose parameters from 1995 to 2000.

³⁴See Saint-Paul and Bentolila (2003).

Moreover, we have $\frac{A_0}{A_1} = 1.38$.³⁵

Finally, we consider the parameters related to offshoring activities. To measure offshoring volume, we use the measure from Feenstra and Hanson (1999) in which offshoring variation is reflected in changes in intermediate inputs imports. They construct the direct measure of the offshoring extent as

$$Off_g = \frac{\sum_h [(intermediates\ purchased\ by\ g\ from\ h) (\frac{imports\ of\ intermediates\ in\ h}{domestic\ consumption\ of\ intermediates\ in\ h})]}{\sum_h (intermediates\ purchased\ by\ g\ from\ h)}$$

Off_g refers to the offshoring extent in the sector g . According to this measure, we can obtain the offshoring extent in the service sector as well as in the manufacturing sector. For example, the offshoring extent in the manufacturing sector I is given as

$$I = \sum_{m \in M} Off_m \frac{C_m}{\sum_{m \in M} C_m}$$

Off_m is the offshoring extent in the sector m that belongs to the set of manufacturing sectors M , the term $\frac{C_m}{\sum_{m \in M} C_m}$ refers to the weight of intermediates consumption in sector m . In our model, the ratio of imported inputs to the total intermediate consumption is equivalent to the proportion of foreign hires, they are $I = \frac{l_{F,Y}}{l_{F,Y} + l_{D,Y}}$ and $J = \frac{l_{F,X}}{l_{F,X} + l_{D,X}}$.

We have five remaining parameters to be determined: c , w^* , τ , β_I and β_J . Because we do not have direct information about the non-task-specific offshoring cost for each sector, we compute them using economy-wide non-task-specific cost of offshoring β , which is equivalent to the transportation cost of trade in other literature. A commonly used value is 1.3 (e.g. Ghironi and Melitz (2005), Felbermayr et al (2011)). Ranjan(2013) adopts a slightly higher value of 1.5 for the calibration exercise because offshoring cost includes more than just transportation cost. In this paper, we adopt the same value as Ranjan (2013). We can also compute the economy-wide offshoring extent, which is 0.305. For simplification, we assume that both sectors have the same task-specific offshoring cost function $t(\cdot)$. Therefore, we can choose β_I and β_J according to the following relationship given the offshoring extent in the manufacturing sector $I = 0.38$ and $J = 0.21$ in the service sector

$$\beta(1 + 0.305)^\tau = \beta_I(1 + I)^\tau = \beta_J(1 + J)^\tau$$

The above relationship is derived from that for the marginal task, there is no difference between hiring workers in the manufacturing sector and hiring workers in the service sector.³⁶ At last, w^* and c are chosen to match the unemployment rate and offshoring extent in the manufacturing sector.³⁷

³⁵In our calibration exercise, we normalize production technology in the manufacturing sector A_0 to 1; therefore, to know A_1 , we simply need to know the ratio of A_0 to A_1 .

³⁶For each marginal task, there is no difference between hiring foreign and domestic workers. Because domestic workers can move freely across sectors and the cost of hiring foreign workers is same for each sector, we can conclude that at the margin, there is no difference between hiring workers in different sectors.

³⁷We can also match the offshoring scale in the service sector.

Parameter	Interpretation	Value	Source
Normalized Parameters			
P	Aggregate price level	1	
A_0	Aggregate productivity parameter	1	
N	Size of labor force	1	
External Parameter Estimates			
r	Monthly discount rate	0.008	Van der Linden et al. (2001)
ϕ	Elasticity of matching function	0.4	Van der Linden et al. (2001)
δ	Job worker separation rate	0.013	Van der Linden et al. (2001)
$\frac{b}{w}$	Unemployment benefit	0.35	OECD (1999)
η	Bargaining power of workers	0.77	Van der Linden et al. (2001)
β	Non-task-specific offshoring cost	1.5	Felbermayr et al (2011)
Parameters Matched to Moments in the Data			
m	Scale parameter of matching function	0.29	Obtained from $u = \frac{\delta}{\delta + m\theta^{0.4}}$ and θ
A_1	Production technology (Serv)	0.72	Estimate by using OECD data
γ_Y	Production function parameter (Manu)	0.59	Estimate by using OECD data
γ_X	Production function parameter (Serv)	0.77	Estimate by using OECD data
ρ	Weight parameter	0.476	Estimate by using OECD $I - O$ data
β_I	Non-task-specific offshoring cost (Manufacture)	Free	match $\beta_I(1 + I)^\tau = \beta_J(1 + J)^\tau = \beta(1 + 0.305)^\tau$
β_J	Non-task-specific offshoring cost (Service)	Free	match $\beta_I(1 + I)^\tau = \beta_J(1 + J)^\tau = \beta(1 + 0.305)^\tau$
c	Vacancy cost	Free	match $u = 0.09, I = 0.38$
w^*	Foreign wage	Free	match $u = 0.09, I = 0.38$

Table 1: Calibration parameter values for Belgium

In the real world, the task-specific offshoring costs would vary across industries, which experience varying difficulty with offshoring tasks.³⁸ Until now, there was no way to identify this task-specific cost function.³⁹ Thus, we are simply going to try two different task-specific offshoring cost functions: one is relatively flat, $t(i) = (1 + i)^2$, and the other is relatively steep, $t(i) = (1 + i)^3$. These cost schedules and simulation methods are also used by Ranjan (2013) in his numerical exercises. For different task-specific cost functions, we obtain different values of w^* and c . We illustrate all the parameter values in Table 1.

³⁸A steeper cost captures an industry where relatively little can be offshored, while a flat cost captures an industry where more tasks can be offshored.

³⁹Grossman and Rossi-Hansberg (2008) take an offshoring cost with form $\frac{1}{(1-i)^\nu}$ as an example, and Ranjan (2013) uses $t(i) = (1 + i)^\nu$ as an example. Both functions are convex.

4.2 Simulation Results

In our simulation exercises, we hold the calibrated values of c and w^* constant at their baseline values for each specification of the task-specific cost function. Here, we are mainly concerned with the impact of offshoring technology progress in the service sector on domestic unemployment and on the extent of offshoring in both the service and the manufacturing sectors. The simulation results are shown in Figures 4 and 5.

Figure 4 shows the results of comparative statics with respect to the offshoring technology (non-task-specific offshoring cost β_J) in the service sector when the task-specific cost function has the form $t(\cdot) = (1 + \cdot)^2$. It is clear that a decrease in the non-task-specific offshoring cost in the service sector leads to an increase in the unemployment rate and the offshoring extent in the service sector, and higher unemployment rate implies lower wage level. As for the impact on the offshoring extent in the manufacturing sector, firms in the manufacturing sector reduce the offshoring scale because hiring domestic workers becomes less costly.

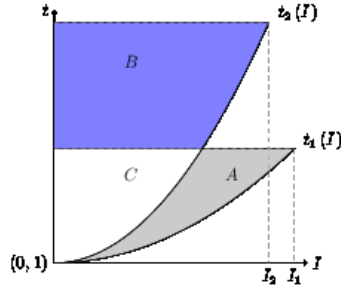


Figure 1: Cost savings for different offshoring cost functions

1

Figure 4: Flat task-specific cost of offshoring $t(i) = (1 + i)^2$

Figure 5 shows the results when the cost function has the form $t(\cdot) = (1 + \cdot)^3$. We repeat the same exercise and can see that a decrease in the non-task-specific offshoring cost in the service sector leads to a decrease in unemployment, thus making it more expensive to hire domestic workers. This, in turn, will enlarge the offshoring extent in the manufacturing sector.

Therefore, the form of the task-specific offshoring cost plays an important role in determining the unemployment trend and the indirect impact on another sector's offshoring activities. For countries that have industries for which offshoring costs vary more across tasks, unemployment is more likely to decrease, and firms in another sectors are more likely to increase their offshoring extent when offshoring technology improves.

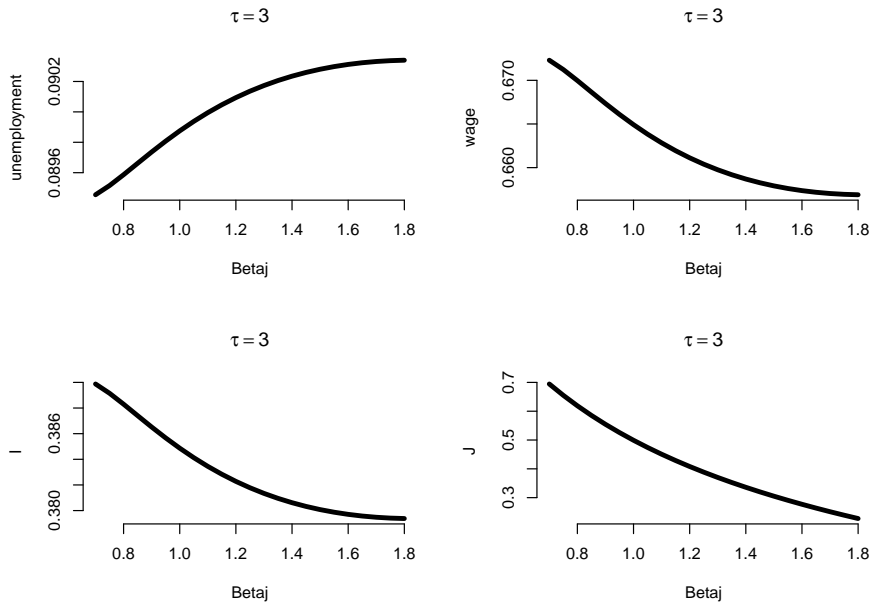


Figure 5: Steep task-specific cost of offshoring $t(i) = (1 + i)^3$

5 Concluding Remarks

This paper examines the impact of increasing offshoring activities in the service sector. It shows that when the marginal task-specific cost of offshoring for trading tasks is sufficiently large, advances in offshoring technology in the service sector reduce domestic unemployment, indirectly encouraging firms in the manufacturing sector to increase the extent of their offshoring activities. This, in turn, enhances the productivity effect of offshoring in the manufacturing sector and may create additional domestic jobs. Based on the theoretical analysis, we calibrate our model using Belgian economic parameters; the simulation results predict varied trends in unemployment and impacts on manufacturing sector offshoring activities when we use different offshoring cost schedules.

References

- [1] Amiti, M., Wei, S., Haskel, J., Auriol, E. (2005a). “Fear of service outsourcing: Is It Justified?” *Economic Policy*, 20, 307-347.
- [2] Amiti, M., Wei, S. (2005b). “Service Offshoring, Productivity, and Employment: Evidence from the United States,” IMF Working Paper WP/05/238.
- [3] Antràs, P., Helpman, E. (2004). “Global Sourcing,” *Journal of Political Economy*, 112(3), 552-580.

- [4] Autor, D., Dorn, D., Hanson, G. (2013). “The China Syndrome: Local Labor Market Effects of Import Competition in the United States,” *American Economic Review*, 103(6), 2121-2168.
- [5] Basco, S., Mestieri, M. (2013). “Heterogeneous Trade Costs and Wage Inequality: A Model of Two Globalizations,” *Journal of International Economics*, 89(2), 393-406.
- [6] Bentolila, S., Saint-Paul, G. (2003). “Explaining Movements in the Labor Share,” *The B.E. Journal of Macroeconomics*, 3(1).
- [7] Beverelli, C., Fiorini, M., Hoekman, B. (2017). “Services Trade Policy and Manufacturing Productivity: The Role of Institutions,” *Journal of International Economics*, 104, 166-182.
- [8] Bhagwati, J. Panagariya, A., Srinivasan, T.N. (2004). “The Muddles over Outsourcing,” *Journal of Economic Perspectives*, 18(4), 93-114.
- [9] Blinder, A. (2006). “Offshoring: The Next Industrial Revolution?” *Foreign Affairs*, 3, 113-128.
- [10] Crinò, R. (2010). “Service Offshoring and White-Collar Employment,” *Review of Economic Studies*, 77(2), 595-632.
- [11] Crinò, R. (2012). “Service Offshoring and the Skill Composition of Labour Demand,” *Oxford Bulletin of Economics and Statistics*, 74(1), 20-57.
- [12] Davidson, C., Matusz, S. (2004). *International Trade and Labor Markets*, WE Upjohn Institute of Employment Research, Kalamazoo, Michigan.
- [13] Dollar, D., Kraay, A. (2002). “Spreading the Wealth,” *Foreign Affairs*, 81(1),120-133.
- [14] Drezner, D. (2004). “The Outsourcing Bogeyman,” *Foreign Affairs*, May/June.
- [15] Felbermayr, G., Prat, J., & Schmerer, H. (2011). “Globalization and Labor Market Outcomes: Wage Bargaining, Search Frictions, and Firm Heterogeneity,” *Journal of Economic Theory*, 146(1), 39-73.
- [16] Freenstra, R.C.(2010). “Offshoring in the Global Economy, Microeconomic Structure and Macroeconomic Implications,” MIT Press, Cambridge, Mass.
- [17] Görg, H., Hanley, A., Strobl, E. (2008). “Productivity Effects of International Outsourcing: Evidence from Plant-Level Data,” *Canadian Journal of Economics*, 41(2),670-688.
- [18] Ghose, A.K. (2000). “Trade Liberalization, Employment, and Global Inequality,” *International Labour Review*, 139(3),281-305.

- [19] Grossman, G., Helpman, E. (2002). "Integration versus Outsourcing in Industry Equilibrium," *Quarterly Journal of Economics* , 117(1), 85-120.
- [20] Grossman, G., Helpman, E. (2005). "Outsourcing in a Global Economy," *Review of Economics Studies*, 72(1), 135-159.
- [21] Groizard, J.L., Ranjan, P., Rodriguez-Lopez, A. (2014). "Offshoring and Jobs: The Myriad Channels of Influence," *European Economic Review*, 72, 221-239.
- [22] Grossman, G. and Rossi-Hansberg, E. (2008). "Trading Tasks: A Simple Theory of Offshoring," *American Economic Review*, 98(5), 1978-1997.
- [23] Hosios, A. (1990). "Factor Market Search and the Structure of Simple General Equilibrium Models," *Journal of Political Economy*, 98(2), 325-355.
- [24] Keuschnigg, C., Ribi, E. (2009). "Outsourcing, unemployment and welfare policy," *Journal of International Economics*, 78(1), 168-176.
- [25] Kohler, W. (2004a). "Aspects of International Fragmentation," *Review of International Economics*, 12(5), 793-816.
- [26] Kohler, W. (2004b). "International Outsourcing and Factor Prices with Multistage Production," *Economic Journal*, 114(494), 166-185.
- [27] Matusz, S. (1994). "International Trade Policy in a Model of Unemployment and Wage Differential," *Canadian Journal of Economics*, 27(4), 939-949.
- [28] Melitz, J. (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695-1725.
- [29] Ghironi, F., Melitz, J. (2005). "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *The Quarterly Journal of Economics*, 120(3), 865-915.
- [30] Michel, B. (2009). "The Impact of Offshoring on Employment in Belgium," Federal Planning Bureau, Working Paper 1-09.
- [31] Mitra, D., Ranjan, P. (2010). "Offshoring and Unemployment: The Role of Search Frictions Labor Mobility," *Journal of International Economics* , 81(2), 219-229.
- [32] Mitra, D., Ranjan, P. (2013). "Fairness, Search Frictions, and Offshoring," *The B.E. Journal of Economic Analysis and Policy*, 13(1), 137-172.
- [33] OECD. (2007). "Offshoring and Employment: Trends and Impacts. Paris."

- [34] Pissarides, C. (2000). *Equilibrium Unemployment Theory*, 2nd ed., Cambridge, MA:MIT Press.
- [35] Ranjan, P. (2013). “Offshoring, Unemployment, and Wages: The Role of Labor Market Institutions,” *Journal of International Economics*, 89(1), 172-186.
- [36] Utar, H., Torres-Ruiz, Luis. (2013). “International Competition and Industrial Evolution: Evidence from the Impact of Chinese Competition on Mexican Maquiladoras,” *Journal of Development Economics*, 105, 267-287.
- [37] Utar, H. (2016). “Workers Beneath the Floodgates: The Impact of Low-Wage Import Competition and Workers’ Adjustment,” CESifo Working Paper Series No. 6224.

6 Appendix

6.1 Maximization Problem of the Firm

The firms' objective function is

$$\begin{aligned} \underset{l_{D,Y}(s), l_{F,Y}(s), V_Y^D(s), V_Y^F(s)}{\text{Max}} \int_t^\infty e^{-r(s-t)} [p_y(s)Y(s) - cV_Y^D(s) - c^F V_Y^F(s) \\ - w_y(s)l_{D,Y}(s) - w^*(s)(l_{F,Y}(s) + l_{D,Y}(s)) \int_0^I \beta_I t(i) di] ds \end{aligned}$$

The dynamics of domestic as well as foreign employed workers in the manufacturing sector are

$$\dot{l}_{D,Y}(t) = q(\theta_Y(t))V_Y^D(t) - \delta l_{D,Y}(t)$$

$$\dot{l}_{F,Y}(t) = V_Y^F(t) - \delta l_{F,Y}(t)$$

The current value Hamiltonian for the firms can be written as

$$\begin{aligned} \mathcal{H} = p_y Y - cV_Y^D - c^F V_Y^F - w_y l_{D,Y} - w^*(l_{F,Y} + l_{D,Y}) \int_0^I \beta_I t(i) di \\ + \zeta_1 (q(\theta_Y) V_Y^D - \delta l_{D,Y}) + \zeta_2 (V_Y^F - \delta l_{F,Y}) \end{aligned}$$

If we only consider the steady state equilibrium, $\dot{\zeta}_1 = 0$, we have the following first order conditions

$$\frac{\partial \mathcal{H}}{\partial V_Y^F} = -c^F + \zeta_2 = -0 + \zeta_2 \quad (23)$$

$$\frac{\partial \mathcal{H}}{\partial V_Y^D} = -c + \zeta_1 q(\theta_Y(t)) = 0 \quad (24)$$

$$\frac{\partial \mathcal{H}}{\partial l_{D,Y}} = p_y \frac{\partial Y}{\partial l_{D,Y}} - w_y - w^* \int_0^I \beta_I t(i) di - w^*(l_{F,Y} + l_{D,Y}) \beta_I t(I) \frac{\partial I}{\partial l_{D,Y}} - \zeta_1 \delta = \zeta_1 r \quad (25)$$

$$\frac{\partial \mathcal{H}}{\partial l_{F,Y}} = p_y \frac{\partial Y}{\partial l_{F,Y}} - w^* \int_0^I \beta_I t(i) di - w^*(l_{F,Y} + l_{D,Y}) \beta_I t(I) \frac{\partial I}{\partial l_{F,Y}} = 0 \quad (26)$$

Combining equation (24) and (25), we have we have

$$w_y + \frac{(r + \delta)c}{q(\theta_Y)} = p_y \frac{\partial Y}{\partial l_{D,Y}} - w^* \int_0^I \beta_I t(i) di - w^*(l_{F,Y} + l_{D,Y}) \beta_I t(I) \frac{\partial I}{\partial l_{D,Y}} \quad (27)$$

Equation (26) over (27) yields

$$w^* \beta_I t(I) = w_y + \frac{(r + \delta)c}{q(\theta_Y)} \quad (28)$$

We plug the equation (28) back into the equation (27) to obtain the job creation condition.⁴⁰ It gives us

$$p_y \frac{\partial Y}{\partial l_{D,Y}} \left[\frac{t(I)}{\int_0^I t(i) di + t(I)(1-I)} \right] = w_y + \frac{(r + \delta)c}{q(\theta_Y)} \quad (29)$$

Then we have two equilibrium conditions, (28) and (29), and two endogenous variables, labor market tightness in the manufacturing sector and offshoring extent.

6.2 Proof of Proposition 1

The term which determines the change in the relative supply of the workers with offshoring activity is $(1 - I)F(I)$. In the closed economy, $I = 0$ and this term becomes 1. The difference between the open economy and the closed economy of this term is given by

$$(1 - I)F(I) - 1 = (1 - I) \left[\frac{t(I)}{\int_0^I t(i) di + t(I)(1 - I)} \right] - 1 \quad (30)$$

Clearly, $(1 - I)F(I) < 1$, and difference is negative.

⁴⁰Since domestic and foreign workers can be perfectly substituted in terms of productivity. Thus we have $p_y \frac{\partial Y}{\partial l_{D,Y}} = p_y \frac{\partial Y}{\partial l_{F,Y}}$.